

Effects of prepared states and Unruh temperature on Measurement-Induced-Nonlocality

Zehua Tian and Jiliang Jing*

*Department of Physics, and Key Laboratory of Low Dimensional
Quantum Structures and Quantum Control of Ministry of Education,
Hunan Normal University, Changsha, Hunan 410081, China*

Abstract

How the prepared states and Unruh effect affect Measurement-Induced-Nonlocality (MIN) is studied. We show that, as the Unruh temperature increases, the MIN between modes A and I decreases but the MIN between modes A and II increases. We prove that the parameters c_i which decide initial prepared states affect not only the values of the MIN, but also the dynamical behavior of it. By comparing the MIN with the maximal expectation values of CHSH inequality and geometric discord between modes A and I, we also find that the MIN is more general than the quantum nonlocality related to violation of Bell's inequalities, and its values is always equal or bigger than that of the geometric discord.

PACS numbers: 03.65.Ud, 03.67.Mn, 04.70.Dy

* Corresponding author, Email: jljing@hunnu.edu.cn

I. INTRODUCTION

The investigation of relativistic quantum information not only supplies the gap of interdisciplinary refer to quantum information and relativity theory, but also has a positive promotion on the development of them. As a result of that, this domain has been paid much attention in the last decade [1–12]. Among them, most papers have focused on quantum resource, e. g., quantum entanglement[1–7, 12] and discord [8, 9], because quantum resource plays an very important role in the quantum information tasks such as teleportation [13] and computation [14, 15], and studying it in a relativistic setting is very closely related to the implementation of quantum tasks with observers in arbitrary relative motion. In addition, extending this work to the black hole background is very helpful for us to understand the entropy and paradox [16, 17] of the black hole.

Despite much effort has been paid to extend quantum information theory to the relativistic setting, another important foundation of quantum mechanics—nonlocality is barely considered. Recently, Nicolai Friis *et al* firstly studied the nonlocality in the noninertial frame, and they pointed out that residual entanglement of accelerated fermions is not non-local [18]. Following them Alexander Smith *et al* studied the tripartite nonlocality in the noninertial frames [19], and DaeKil Park considered tripartite entanglement-dependence of tripartite nonlocality [20]. Generally, most researchers analyzed the quantum nonlocality by means of Bell’s inequalities [21] for bipartite system and Svetlichny inequality for tripartite system [22], respectively. Because, these inequalities are satisfied by any local hidden variable theory, but they may be violated by quantum mechanics. However, Shunlong Luo and Shuangshuang Fu have introduced a new way to quantify nonlocality by measurement, which is called the MIN [23], and following their paper, a number of papers emerged to perfect its definition [24, 25] and discussed its properties [26, 27]. In addition, some authors have analyzed its dynamical behavior and compared it with other quantum correlation measurements such as the geometric discord [28, 29]. However, all of these studies don’t involve the effect on the MIN resulting from relativistic effect. In fact, the study that how the Unruh effect [30] affects the MIN not only is very significant to theory study, but also plays a key role in practice, which can help us to implement the quantum task preferably and more efficiently. Inspired by that, we analyze how the Unruh effect and prepared states affect the MIN in this paper and we will show some new properties.

Our paper is constructed as follows. In section II we introduce the different vacuums for relativistic observers and the definition of the MIN. In section III how the Unruh effect and prepared states affect the MIN are studied. And in the last section we summarize and discuss our conclusions.

II. VACUUMS, EXCITED STATES AND DEFINITION OF MIN

It is well known that an uniformly accelerated observer will detect a thermal particle distribution in the Minkowski vacuum. The Minkowski vacuum can be factorized as a product of the vacua of all different Unruh modes

$$|0\rangle_{\text{M}} = \otimes_w |0_w\rangle_{\text{U}}. \quad (1)$$

For the Dirac field, the Unruh monochromatic mode $|0_w\rangle_{\text{U}}$, from the noninertial observers' perspective, can be expressed as [2, 7, 8]

$$|0_w\rangle_{\text{U}} = (e^{-w/T} + 1)^{-\frac{1}{2}} |0_w\rangle_{\text{I}} |0_w\rangle_{\text{II}} + (e^{w/T} + 1)^{-\frac{1}{2}} |1_w\rangle_{\text{I}} |1_w\rangle_{\text{II}}, \quad (2)$$

where $|m\rangle_{\text{I}}$ ($|n\rangle_{\text{II}}$) denote Rindler mode in region I (region II), and $T = a/2\pi$ is the Unruh temperature in which a denotes the proper acceleration of the noninertial observer. Likewise, the particle state of Unruh mode w in the Rindler basis is found to be

$$|1_w\rangle_{\text{U}} = |1_w\rangle_{\text{I}} |0_w\rangle_{\text{II}}. \quad (3)$$

Recently, Luo *et al* [23] have introduced a way to quantify nonlocality from a geometric perspective in terms of measurements, which is named the MIN. For a bipartite quantum state ρ shared by subsystem A and B with respective to Hilbert space H^A and H^B , we can find the difference between the overall pre-measurement and post-measurement states by performing a local von Neumann measurements on part A . To capture the genuine nonlocal effect of the measurements on the state, the key point is that the measurements do not disturb the local state $\rho^A = \text{tr}_B \rho$. Based on this idea, the MIN can be defined by *et al* [23]

$$N(\rho) = \max_{\Pi^A} \| \rho - \Pi^A(\rho) \|^2. \quad (4)$$

For a general 2×2 dimensional system

$$\rho = \frac{1}{2} \frac{\mathbf{1}^A}{\sqrt{2}} \otimes \frac{\mathbf{1}^B}{\sqrt{2}} + \sum_{i=1}^3 x_i X_i \otimes \frac{\mathbf{1}^B}{\sqrt{2}} + \frac{\mathbf{1}^A}{\sqrt{2}} \otimes \sum_{j=1}^3 y_j Y_j + \sum_{i=1}^3 \sum_{j=1}^3 t_{ij} X_i \otimes Y_j, \quad (5)$$

its MIN is given by [23]

$$N(\rho) = \begin{cases} \text{tr}TT^t - \frac{1}{\|\mathbf{x}\|^2}\mathbf{x}^tTT^t\mathbf{x} & \text{if } \mathbf{x} \neq 0, \\ \text{tr}TT^t - \lambda_3 & \text{if } \mathbf{x} = 0, \end{cases} \quad (6)$$

where TT^t ($T = (t_{ij})$) is a 3×3 dimensional matrix, λ_3 is its minimum eigenvalue, and $\|\mathbf{x}\|^2 = \sum_i x_i^2$ with $\mathbf{x} = (x_1, x_2, x_3)^t$.

III. MIN OF X-TYPE INITIAL STATES

We now assume that Alice and Rob share a X-type initial state

$$\rho_{AB} = \frac{1}{4} \left(I_{AB} + \sum_{i=1}^3 c_i \sigma_i^{(A)} \otimes \sigma_i^{(B)} \right), \quad (7)$$

where $I_{A(B)}$ is the identity operator in subspace $A(B)$, and $\sigma_i^{(n)}$ is the Pauli operator in direction i acting on the subspace $n = A, B$, $c_i \in \Re$ such that $0 \leq |c_i| \leq 1$ for $i = 1, 2, 3$. Obviously, Eq. (7) represents a class of states including the well-known initial states, such as the Werner initial state ($|c_1| = |c_2| = |c_3| = \alpha$), and Bell basis state ($|c_1| = |c_2| = |c_3| = 1$).

After the coincidence of Alice and Rob, Alice stays stationary while Rob moves with an uniform acceleration a . To describe the states shared by these two relatively accelerated observers in detail, we must use Eqs.(2) and (3) to rewrite Eq.(7) in terms of Minkowski modes for Alice, Rindler modes I for Rob and Rindler modes II for Anti-Rob, which implies that Rob and Anti-Rob are respectively confined in region I and II. The regions I and II are causally disconnected, and the information which is physically accessible to the observers is encoded in the Minkowski modes A and Rindler modes I, but the physically inaccessible information is encoded in the Minkowski modes A and Rindler modes II. So we must trace over the Rindler modes II (modes I) when we only consider the Physically accessible (unaccessible) information.

A. MIN shared by Alice and Rob

We first consider the MIN between modes A and I . By taking the trace over the states of region II , we obtain

$$\rho_{A,I} = \frac{1}{4} \begin{pmatrix} \frac{1+c_3}{e^{-w/T}+1} & 0 & 0 & \frac{c_1-c_2}{(e^{-w/T}+1)^{\frac{1}{2}}} \\ 0 & (1-c_3) + \frac{1+c_3}{e^{w/T}+1} & \frac{c_1+c_2}{(e^{-w/T}+1)^{\frac{1}{2}}} & 0 \\ 0 & \frac{c_1+c_2}{(e^{-w/T}+1)^{\frac{1}{2}}} & \frac{1-c_3}{e^{-w/T}+1} & 0 \\ \frac{c_1-c_2}{(e^{-w/T}+1)^{\frac{1}{2}}} & 0 & 0 & (1+c_3) + \frac{1-c_3}{e^{w/T}+1} \end{pmatrix},$$

where $|mn\rangle = |m\rangle_A |n\rangle_I$. For convenience to calculate the MIN, we rewrite the state $\rho_{A,I}$ in terms of Bloch representation, which is given by

$$\rho_{A,I} = \frac{1}{4} \left(\mathbf{1}_A \otimes \mathbf{1}_I + c'_0 \mathbf{1}_A \otimes \sigma_3^{(I)} + \sum_{i=1}^3 c'_i \sigma_i^{(A)} \otimes \sigma_i^{(I)} \right), \quad (8)$$

where $c'_0 = \frac{-1}{(e^{w/T}+1)}$, $c'_1 = \frac{c_1}{(e^{-w/T}+1)^{\frac{1}{2}}}$, $c'_2 = \frac{c_2}{(e^{-w/T}+1)^{\frac{1}{2}}}$ and $c'_3 = \frac{c_3}{(e^{-w/T}+1)}$. From Eq.(6), we find that the MIN for the state $\rho_{A,I}$ is

$$N(\rho_{A,I}) = \frac{1}{4} \left\{ \frac{(c_1)^2}{(e^{-w/T}+1)} + \frac{(c_2)^2}{(e^{-w/T}+1)} + \frac{(c_3)^2}{(e^{-w/T}+1)^2} - \min \left[\frac{(c_1)^2}{(e^{-w/T}+1)}, \frac{(c_2)^2}{(e^{-w/T}+1)}, \frac{(c_3)^2}{(e^{-w/T}+1)^2} \right] \right\}. \quad (9)$$

Obviously, $\min \left[\frac{(c_1)^2}{(e^{-w/T}+1)}, \frac{(c_2)^2}{(e^{-w/T}+1)}, \frac{(c_3)^2}{(e^{-w/T}+1)^2} \right]$ depends on both the coefficients c_i of the states in Eq.(7) and the Unruh temperature.

(i) If $|c_1|, |c_2| \geq |c_3|$ in Eq.(7), the minimum term in Eq.(9) is $\frac{(c_3)^2}{(e^{-w/T}+1)^2}$. In this case, the MIN, provided taking fixed c_i , decreases monotonously as the Unruh temperature increases.

(ii) For the case of $|c_3| > \min\{|c_1|, |c_2|\}$ and both c_1 and c_2 don't equals to 0 at the same time, if $\min\{|c_1|, |c_2|\} \geq \frac{\sqrt{2}}{2}|c_3|$, the MIN has a peculiar dynamics with a sudden change as the Unruh temperature increases, i.e., $N(\rho_{A,I})$ decays quickly until

$$T_{sc} = \frac{-w}{\ln \left(\frac{|c_3|^2}{\min\{|c_1|^2, |c_2|^2\}} - 1 \right)}, \quad (10)$$

and then $N(\rho_{A,I})$ decays relatively slowly. Otherwise, the MIN decays monotonously as the temperature increases.

(iii) Finally, if $|c_1| = |c_2| = 0$, we have a monotonic decay of $N(\rho_{A,I})$ as the temperature increases.

The decrease of MIN means that the difference between the pre- and post-measurement states becomes smaller, i.e., the disturbance induced by local measurement weakens. If we understand the MIN as some kind of correlations, this decrease means that the quantum correlation shared by two relatively accelerated observers decreases, i.e., less quantum resource can be used for the quantum information task by these two observers. So the Unruh effect affects quantum communication process by inducing the decrease of quantum resource.

By taking $w = 1$, the dynamical behavior of $N(\rho_{A,I})$ is shown in Fig. 1. We find from the figure that the MIN, as the Unruh temperature approaches to the infinite, has a limit

$$\lim_{T \rightarrow \infty} N(\rho_{A,I}) = \frac{1}{16} \{2(c_1)^2 + 2(c_2)^2 + (c_3)^2 - \min[2(c_1)^2, 2(c_2)^2, (c_3)^2]\}. \quad (11)$$

That is to say, as long as the initial MIN does not equal to zero, it can persist for arbitrary Unruh temperature.

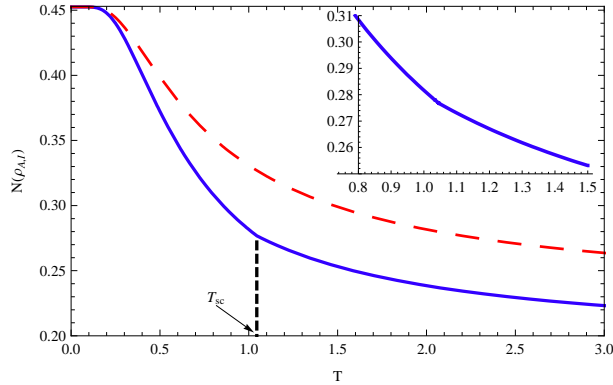


FIG. 1: (Color online) The MIN of state $\rho_{A,I}$ as a function of Unruh temperature T . We take parameters $c_1 = 1$, $c_2 = 0.9$ and $|c_3| \leq |c_1|, |c_2|$ for red dashed line; $c_1 = 0.9$, $c_2 = 0.85$ and $c_3 = 1$ for blue solid line. The insert shows the detail of sudden change.

We study T_{sc} of Eq.(10), if $|c_1| \leq |c_2|$, by taking fixed c_3 , we plot how the parameter c_1 affects it in Fig. 2, which shows that it decreases monotonously as c_1 increases. That is to say, the bigger c_1 is, the sudden change behavior occurs earlier. And when $|c_2| \leq |c_1|$, it is interesting to note that with the increase of c_2 , it decreases monotonously too.

In Fig. 3, we study how the prepared states affect the MIN for case (i). It is found that the $N(\rho_{A,I})$ increases monotonously as $|c_i|$ ($i=1,2$) increases. And for the case that $|c_3| > \min\{|c_1|, |c_2|\} \geq \frac{\sqrt{2}}{2}|c_3|$, the MIN depends only on $|c_3|$ and $\max\{|c_1|, |c_2|\}$ before T_{sc} ,

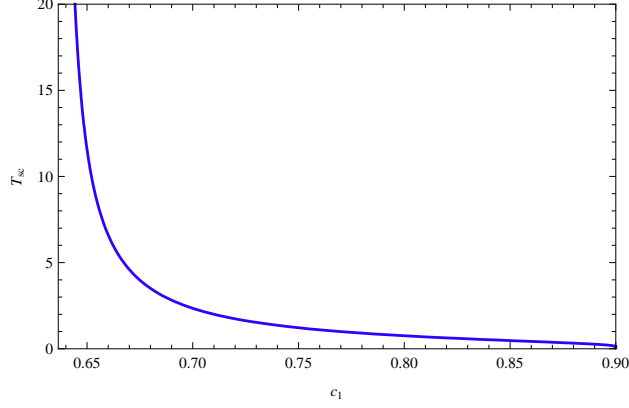


FIG. 2: (Color online) The T_{sc} as a function of c_1 , here we take $|c_1| \leq |c_2|$ and $c_3 = 0.9$.

while after T_{sc} it is independent of $|c_3|$ but dependent of $|c_1|$, $|c_2|$. However, no matter which case, the MIN increases with the increase of $|c_i|$ it depends on.

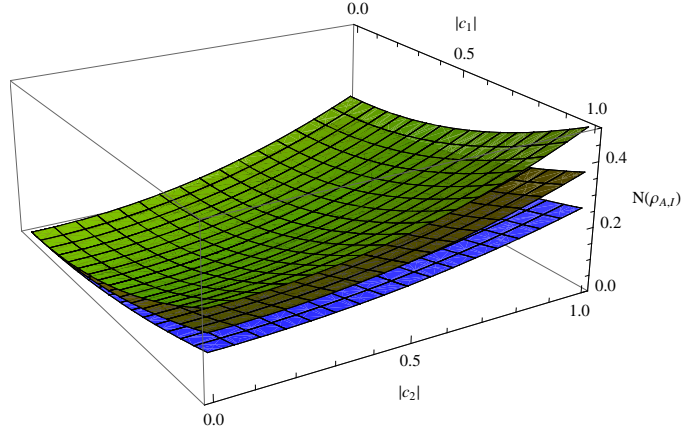


FIG. 3: (Color online) The MIN of state $\rho_{A,I}$ as function of $|c_1|$ and $|c_2|$ with $|c_1|, |c_2| \geq |c_3|$. Here we take fixed $T = 0.1, 1, 20$ from top to bottom, respectively.

Because the MIN is introduced to describe non-locality, its definition is very similar to that of geometric discord. For further understanding it, we will compare it with the maximal possible value $\langle B_{max} \rangle$ of the Bell-CHSH expectation value and geometric discord.

As shown in Ref.[18], the $\langle B_{max} \rangle$ for a given state ρ is determined by

$$\langle B_{max} \rangle_{\rho} = 2\sqrt{\mu_1 + \mu_2}, \quad (12)$$

where μ_1, μ_2 are the two largest eigenvalues of $U(\rho) = TT^t$, the matrix $T = (t_{ij})$ with

$t_{ij} = \text{Tr}[\rho\sigma_i \otimes \sigma_j]$. And the geometric discord is defined as [31, 32]

$$D(\rho) = \frac{1}{4}(\|\vec{x}\|^2 + \|T\|^2 - k_{\max}), \quad (13)$$

where k_{\max} is the largest eigenvalue of matrix $K = \vec{x}\vec{x}^t + TT^t$, where $\vec{x} = (x_i)^t$ with $x_i = \text{Tr}[\rho\sigma_i \otimes \mathbf{1}]$ and T have the same definitions with Eq.(12).

Using Eqs.(8), (12) and (13), $\langle B_{\max} \rangle_{\rho_{A,I}}$ and $D(\rho_{A,I})$ are given by

$$\begin{aligned} \langle B_{\max} \rangle_{\rho_{A,I}} = & 2\left\{ \frac{(c_1)^2}{(e^{-w/T} + 1)} + \frac{(c_2)^2}{(e^{-w/T} + 1)} + \frac{(c_3)^2}{(e^{-w/T} + 1)^2} \right. \\ & \left. - \min\left[\frac{(c_1)^2}{(e^{-w/T} + 1)}, \frac{(c_2)^2}{(e^{-w/T} + 1)}, \frac{(c_3)^2}{(e^{-w/T} + 1)^2} \right] \right\}^{1/2}, \end{aligned} \quad (14)$$

and

$$\begin{aligned} D(\rho_{A,I}) = & \frac{1}{4}\left\{ \frac{(c_1)^2}{(e^{-w/T} + 1)} + \frac{(c_2)^2}{(e^{-w/T} + 1)} + \frac{(c_3)^2}{(e^{-w/T} + 1)^2} \right. \\ & \left. - \max\left[\frac{(c_1)^2}{(e^{-w/T} + 1)}, \frac{(c_2)^2}{(e^{-w/T} + 1)}, \frac{(c_3)^2}{(e^{-w/T} + 1)^2} \right] \right\}, \end{aligned} \quad (15)$$

respectively.

It is interesting to note that

$$N(\rho_{A,I}) = \frac{1}{16} \langle B_{\max} \rangle_{\rho_{A,I}}^2. \quad (16)$$

We plot $N(\rho_{A,I})$ versus $\langle B_{\max} \rangle_{\rho_{A,I}}$ in Fig.4, which shows that $N(\rho_{A,I})$ increases monotonously as $\langle B_{\max} \rangle_{\rho_{A,I}}$ increases and it vanishes at zero point of $\langle B_{\max} \rangle_{\rho_{A,I}}$. It is well known that Bell inequality must be obeyed by local realism theory, but may be violated by quantum mechanics. If we get $\langle B_{\max} \rangle_{\rho_{A,I}} > 2$, it means that the violation of Bell-CHSH inequality, which tells us that there exists nonlocal quantum correlation. But when $\langle B_{\max} \rangle_{\rho_{A,I}} \leq 2$, it doesn't mean that no quantum correlation exists, at least for some mixed states, which have quantum correlation but obey the Bell inequality. So we can't be sure that whether quantum correlation exists or not when $\langle B_{\max} \rangle_{\rho_{A,I}} \leq 2$. However, the MIN, which is an indicator of the global effect caused by locally invariant measurement, is introduced to quantify nonlocality, and nonzero MIN means existence of nonlocality. And from Fig.4 we see that the MIN persists for all $\langle B_{\max} \rangle_{\rho_{A,I}}$ except for zero. Thus, the MIN, understood as some kind of correlations, is more general than the quantum nonlocality related to violation of the Bell's inequalities.

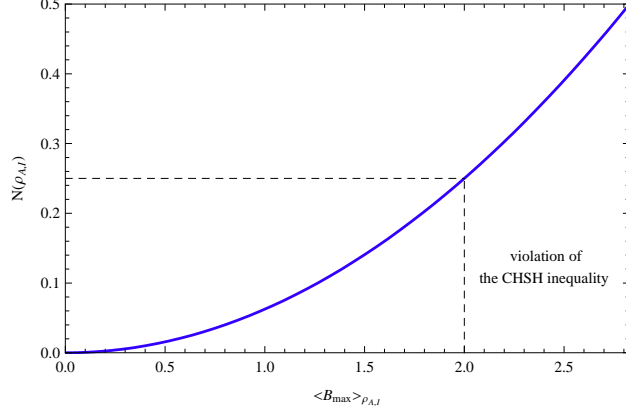


FIG. 4: (Color online) The MIN of state $\rho_{A,I}$ as function of the maximally possible value of the Bell-CHSH expectation value.

From Eqs.(9) and (15), we can see that the MIN is proportional to the two largest eigenvalues of the matrix TT^t , while the geometric discord is proportional to the two smallest eigenvalues of it, so we know that the MIN should be always equal or larger than the geometric discord. In Fig. 5 we plot the MIN versus the geometric discord for the Werner ($|c_1| = |c_2| = |c_3| = c$) states. It is shown that the MIN increases monotonously as the geometric discord increases, and it is always equal or larger than the geometric discord. So as the quantum resource, the MIN is more robust than geometric discord.

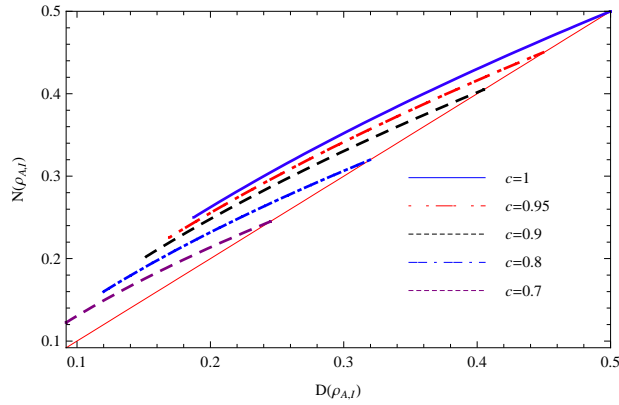


FIG. 5: (Color online) The MIN of state $\rho_{A,I}$ as a function of geometric discord $D(\rho_{A,I})$ for the Werner states, and the red solid line represents $N(\rho_{A,I}) = D(\rho_{A,I})$.

B. MIN shared by Alice and Anti-Rob

Now we consider the MIN between modes A and II. By tracing over all modes in region I, we get

$$\rho_{A,II} = \frac{1}{4} \left(\mathbf{1}_A \otimes \mathbf{1}_{II} + c'_0 \mathbf{1}_A \otimes \sigma_3^{(II)} + \sum_{i=1}^3 c'_i \sigma_i^{(A)} \otimes \sigma_i^{(II)} \right), \quad (17)$$

where $c'_0 = \frac{1}{(e^{-w/T}+1)}$, $c'_1 = \frac{c_1}{(e^{w/T}+1)^{\frac{1}{2}}}$, $c'_2 = \frac{-c_2}{(e^{w/T}+1)^{\frac{1}{2}}}$ and $c'_3 = \frac{-c_3}{(e^{w/T}+1)}$. Similarly, the MIN of state $\rho_{A,II}$ can be obtained according to Eq.(6), which is

$$N(\rho_{A,II}) = \frac{1}{4} \left\{ \frac{(c_1)^2}{(e^{w/T}+1)} + \frac{(c_2)^2}{(e^{w/T}+1)} + \frac{(c_3)^2}{(e^{w/T}+1)^2} \right. \\ \left. - \min \left[\frac{(c_1)^2}{(e^{w/T}+1)}, \frac{(c_2)^2}{(e^{w/T}+1)}, \frac{(c_3)^2}{(e^{w/T}+1)^2} \right] \right\}. \quad (18)$$

(i) If $|c_1|, |c_2| \geq |c_3|$, the MIN increases monotonously as the Unruh temperature increases provided taking fixed c_i .

(ii) For the case of $|c_3| > \min\{|c_1|, |c_2|\}$ and both c_1 and c_2 don't equal to 0 at the same time, if $\min\{|c_1|, |c_2|\} \leq \frac{\sqrt{2}}{2}|c_3|$, the MIN has a peculiar dynamics with a sudden change at T_{sc}

$$T_{sc} = \frac{w}{\ln \left(\frac{|c_3|^2}{\min\{|c_1|^2, |c_2|^2\}} - 1 \right)}. \quad (19)$$

Otherwise, the MIN increases monotonously with the increase of the Unruh temperature.

(iii) Finally, if $|c_1| = |c_2| = 0$, we have a monotonic increase of $N(\rho_{A,II})$ as the Unruh temperature increases.

We plot $N(\rho_{A,II})$ versus the Unruh temperature in Fig. 6. It is found that the MIN, as the Unruh temperature approaches to the infinite, is close to

$$\lim_{T \rightarrow \infty} N(\rho_{A,II}) = \frac{1}{16} \{ 2(c_1)^2 + 2(c_2)^2 + (c_3)^2 - \min[2(c_1)^2, 2(c_2)^2, (c_3)^2] \}, \quad (20)$$

which is the same as $\lim_{T \rightarrow \infty} N(\rho_{A,I})$. In addition, as $T = 0$ the MIN vanishes, which means that the correlation between A and II is local when the observers are inertial.

When $|c_1| < |c_2|$, by taking fixed c_3 , we plot T_{sc} as a function of c_1 in Fig. 7. We learn from the figure that, unlike the Fig.2, T_{sc} increases monotonously with the increase of c_1 . That is to say, the bigger c_1 is, the sudden change behavior occurs latter. And when $|c_2| < |c_1|$, it is also important to note that as $|c_2|$ increases T_{sc} increases monotonously too.

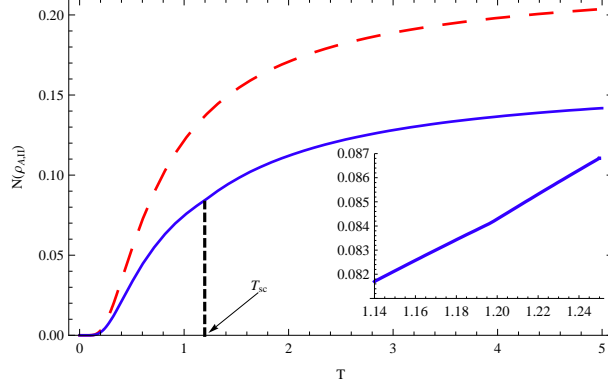


FIG. 6: (Color online) The MIN of state $\rho_{A,II}$ as a function of Unruh temperature T . We take parameters $c_1 = 1$, $c_2 = 0.9$ and $|c_3| \leq |c_1|, |c_2|$ for red dashed line; $c_1 = 0.9, c_2 = 0.55$ and $c_3 = 1$ for blue solid line. The insert shows the detail of sudden change.

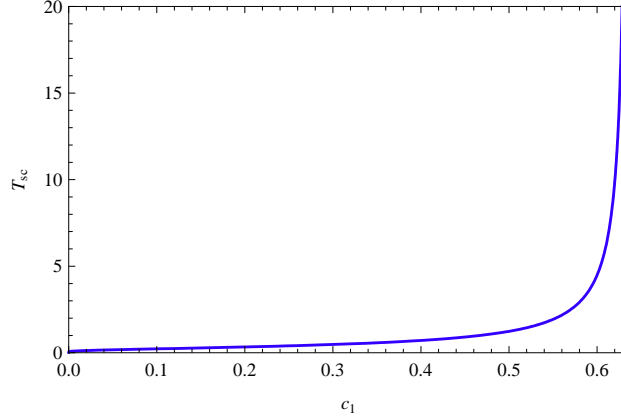


FIG. 7: (Color online) The T_{sc} as a function of c_1 , here we take $|c_1| \leq |c_2|$ and $c_3 = 0.9$.

How the prepared states affect the MIN for case (i) is shown in Fig. 8, which tells us that $N(\rho_{A,II})$ increases monotonously as $|c_i|$ ($i = 1, 2$) increases. And for the case that $|c_3| > \min\{|c_1|, |c_2|\}$, the MIN is independent of $|c_3|$ but dependent of $|c_1|, |c_2|$ before T_{sc} , while after T_{sc} it depends on $|c_3|$ and $\max\{|c_1|, |c_2|\}$. However, no matter which case, the MIN increases with the increase of $|c_i|$.

From the above discussions, we know that the Unruh effect can induce the degradation for $N(\rho_{A,I})$, but the increase for $N(\rho_{A,II})$. However, $N(\rho_{A,I}) + N(\rho_{A,II})$ has different dynamics for different classes of states: (i) When $|c_1|, |c_2| \geq |c_3|$, $N(\rho_{A,I}) + N(\rho_{A,II})$ is independent of the Unruh temperature. That is to say, $N(\rho_{A,I}) + N(\rho_{A,II})$ is a constant versus the Unruh temperature for this class of states; (ii) When $|c_3| > \min\{|c_1|, |c_2|\} \geq \frac{\sqrt{2}}{2}|c_3|$, with

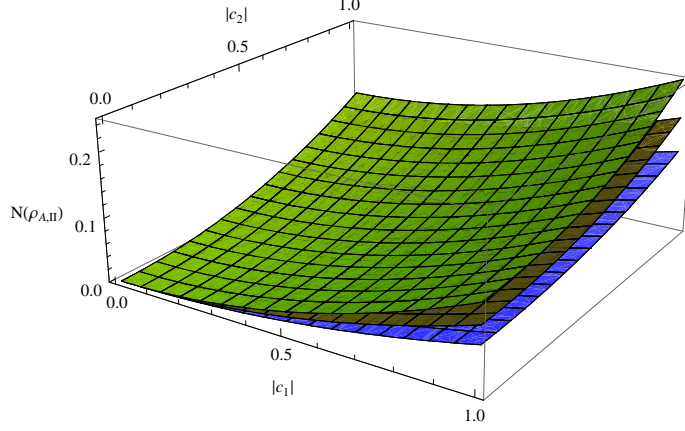


FIG. 8: (color online) The MIN of state $\rho_{A,II}$ as function of c_1 and c_2 with $|c_1|, |c_2| \geq |c_3|$. Here we take fixed $T = \infty, 2, 1$ from top to bottom, respectively.

the increase of the Unruh temperature $N(\rho_{A,I}) + N(\rho_{A,II})$ decreases monotonously until

$$T_{sc} = \frac{-w}{\ln\left(\frac{|c_3|^2}{\min\{|c_1|^2, |c_2|^2\}} - 1\right)}, \quad (21)$$

and from then on it remains constant; And (iii) when $\min\{|c_1|, |c_2|\} < \frac{\sqrt{2}}{2}|c_3|$, $N(\rho_{A,I}) + N(\rho_{A,II})$ decays quickly until

$$T_{sc} = \frac{w}{\ln\left(\frac{|c_3|^2}{\min\{|c_1|^2, |c_2|^2\}} - 1\right)}, \quad (22)$$

and after that it decays relatively slowly. We plot these dynamical behaviors in Fig. 9.

IV. CONCLUSIONS

The effect of the prepared states and Unruh temperature on the MIN of Dirac fields was investigated and the following new properties were found: (i) The MIN $N(\rho_{A,I})$ for the X-type states decreases as the Unruh temperature increases, but $N(\rho_{A,II})$ increases with the increase of the Unruh temperature. (ii) For fixed Unruh temperature, it is found that the MIN always increases as $|c_i|$ ($i = 1, 2, 3$) increases, and it takes the maximal value for the Bell basis states. (iii) Both $N(\rho_{A,I})$ and $N(\rho_{A,II})$ have a peculiar dynamics with a sudden change at T_{sc} provided c_i appropriately chosen, the T_{sc} for $N(\rho_{A,I})$ decreases as c_i increases, while it is contrary for $N(\rho_{A,II})$. (iv) The MIN is more general than the quantum nonlocality related to violation of Bell's inequalities. Besides, it increases as the geometric

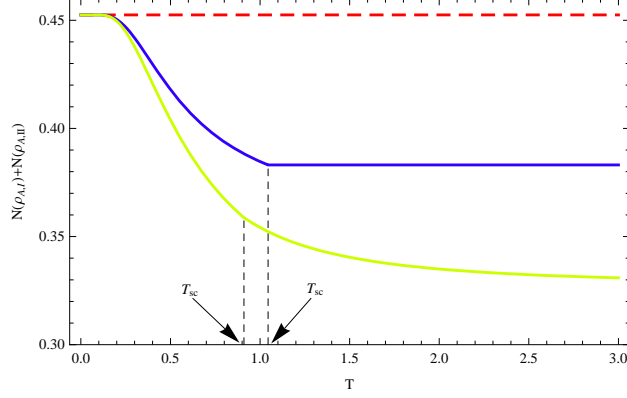


FIG. 9: (Color online) The sum of $N(\rho_{A,I})$ and $N(\rho_{A,II})$ as a function of the Unruh temperature. We take $c_1 = 1, c_2 = 0.9$ and $|c_3| \leq |c_1|, |c_2|$ for the red dashed line; $c_3 = 1, c_1 = 0.9$ and $c_2 = 0.85$ for the blue solid line, and $c_3 = 1, c_1 = 0.9$ and $c_2 = 0.5$ for the yellow solid line.

discord increases, and it is always equal or larger than the geometric discord. And (v) $N(\rho_{A,I}) + N(\rho_{A,II})$ has three kinds of dynamics: (a) When $|c_1|, |c_2| \geq |c_3|$, it is independent of the Unruh temperature; (b) When $|c_3| > \min\{|c_1|, |c_2|\} \geq \frac{\sqrt{2}}{2}|c_3|$, with the increase of the Unruh temperature it decreases monotonously until $T_{sc} = \frac{-w}{\ln(\frac{|c_3|^2}{\min\{|c_1|^2, |c_2|^2\}} - 1)}$, and from then on it remains constant; And (c) when $\min\{|c_1|, |c_2|\} < \frac{\sqrt{2}}{2}|c_3|$, with the increase of the Unruh temperature, it decays quickly until $T_{sc} = \frac{w}{\ln(\frac{|c_3|^2}{\min\{|c_1|^2, |c_2|^2\}} - 1)}$, and after that it decays relatively slowly.

Here we just simply discuss the relation between the MIN and the maximal expectation values of CHSH inequality and geometric discord. More detailed study of this relation can help us to not only understand the MIN more clearly, but also distinguish difference of quantum resource based on different correlation measurements. Such topics are left for future research.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 11175065, 10935013; the National Basic Research of China under Grant No. 2010CB833004; the SRFDP under Grant No. 20114306110003; PCSIRT, No. IRT0964; the Hunan Provincial Natural Science Foundation of China under Grant No 11JJ7001; and

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